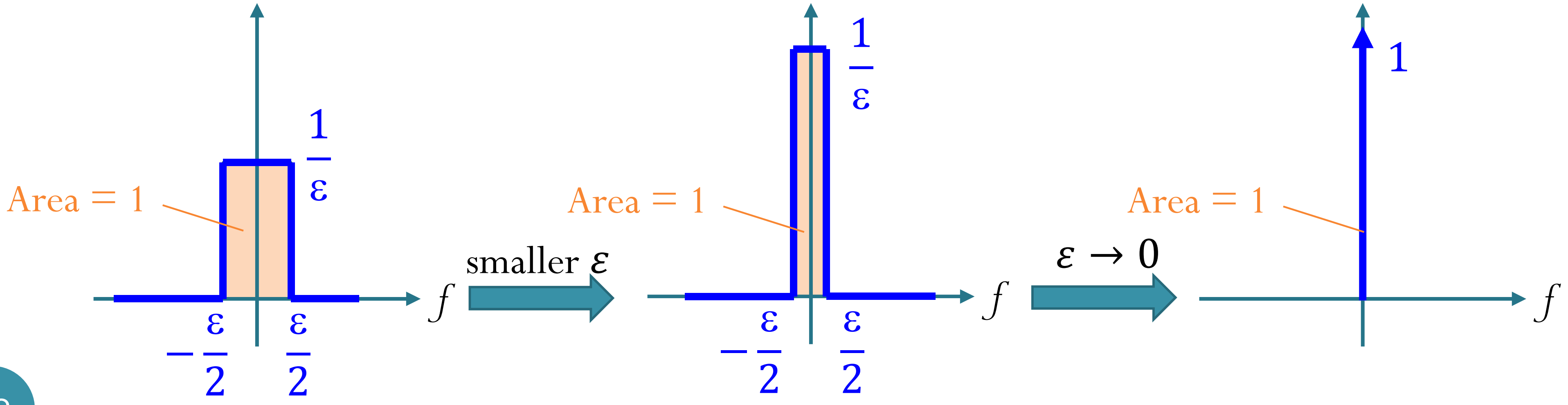
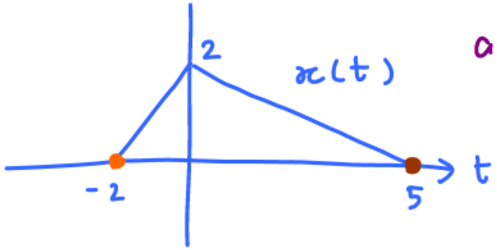
Delta function δ(*f*)

- (Dirac) delta function or (unit) impulse function
- Usually depicted as a vertical arrow at the origin
- Not a true function
  - Undefined at $f = 0$
- Intuitively we may visualize δ(*f*) as an infinitely tall, infinitely narrow rectangular pulse of **unit area**

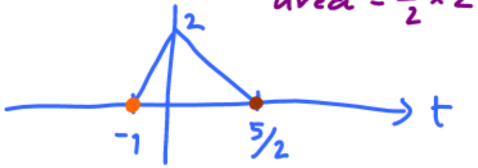


Time Manipulation



area: $\frac{1}{2} \times 2 \times (5-(-2))$

$x(\underline{\underline{2}}t)$



area $= \frac{1}{2} \times 2 \times (5-(-2)) \times \frac{1}{2}$

- Consider a function of time $x(t)$.
- **Time shifting**:
  - When $\boldsymbol{T > 0}$, $x(t-T)$ is $x(t)$ right-shifted (**delayed**) by T.
  - When $\boldsymbol{T < 0}$, $x(t-T)$ is $x(t)$ left-shifted (**advanced**) by $|T|$.
  - Summary: $g(t-T)$ is $g(t)$ right-shifted by T.

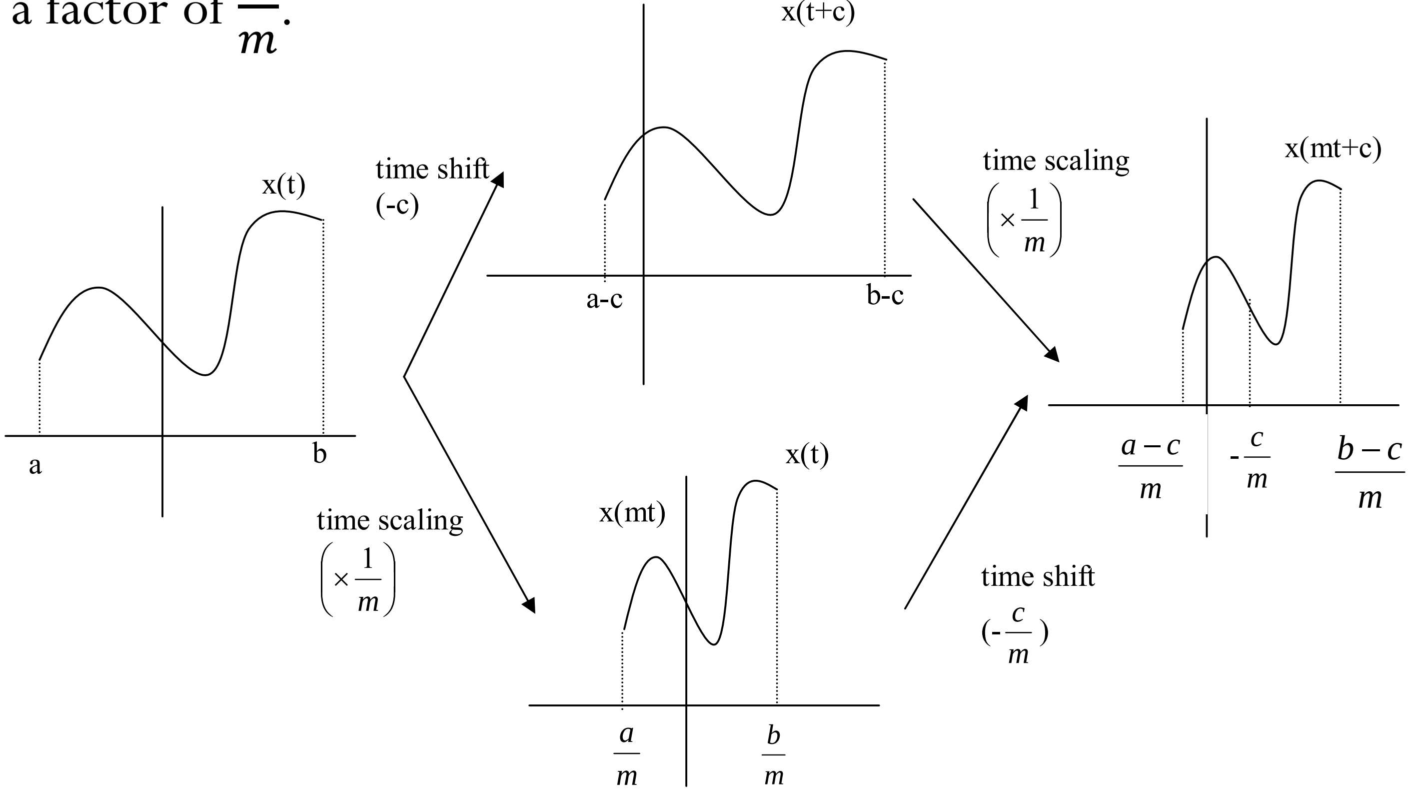
$x(\frac{1}{2}t)$

- **Time scaling**:
  - When $\boldsymbol{0 < a < 1}$, $x(at)$ is $x(t)$ **expanded** in time by a factor of $\frac{1}{a}$.
  - When $\boldsymbol{a > 1}$, $x(at)$ is $x(t)$ **compressed** in time by a factor of a.
  - Summary: When $\boldsymbol{a > 0}$, $x(at)$ is $x(t)$ **scaled** horizontally by a factor of $\frac{1}{a}$.
  - Note that the signal remains anchors at $t = 0$. In other words, the signal at $t = 0$ remains unchanged.
- **Time inversion** (or folding):
  - $x(-t)$ is the mirror image of $x(t)$ about the vertical axis.

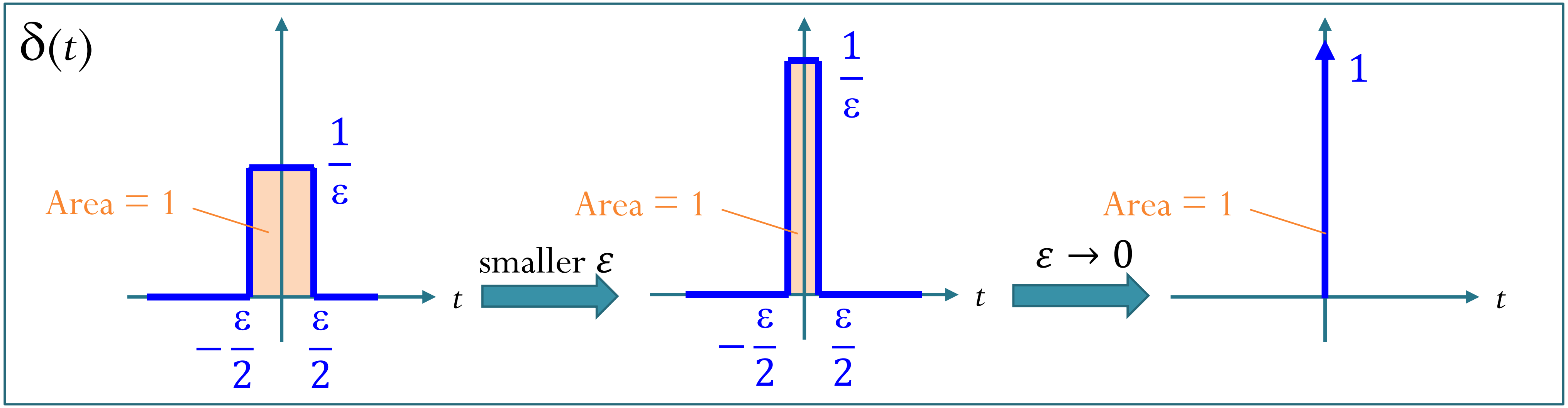
[Lathi & Ding, 2009, Section 2.3, p. 28-32]

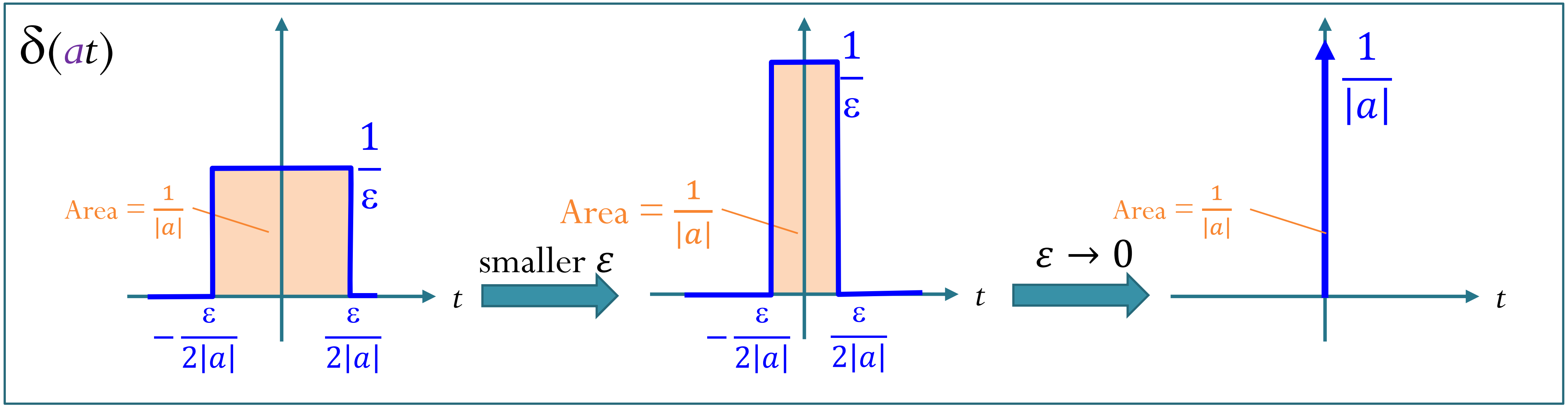
Time Manipulation

- For $x(mt + c)$, may consider it as
  - $x\left(m\left(t - \left(-\frac{c}{m}\right)\right)\right)$: First scale $x(t)$ horizontally by a factor of $\frac{1}{m}$. Then, right-shift by $-\frac{c}{m}$.
  - $x\big((mt) - (-c)\big)$: First right-shift $x(t)$ by $-\frac{c}{m}$. Then scale horizontally by a factor of $\frac{1}{m}$.



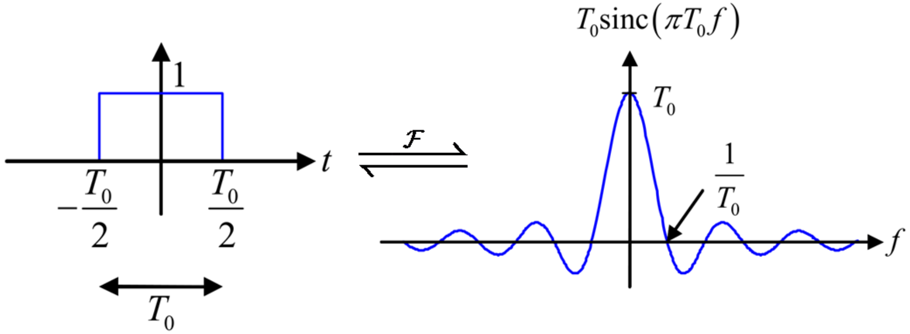
$\delta(at)$

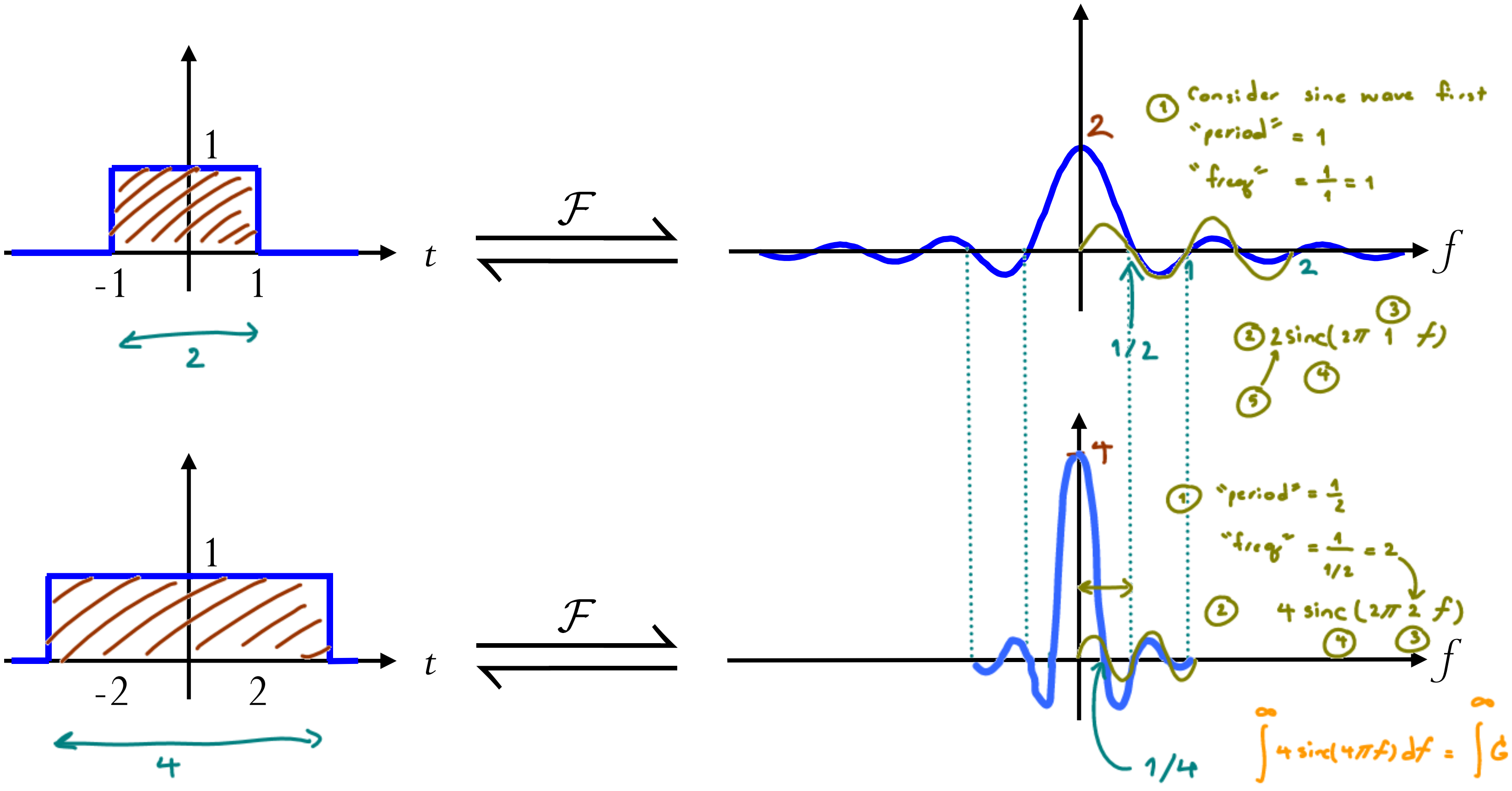




Here, I use $a = \frac{1}{2}$.

Practice Problems





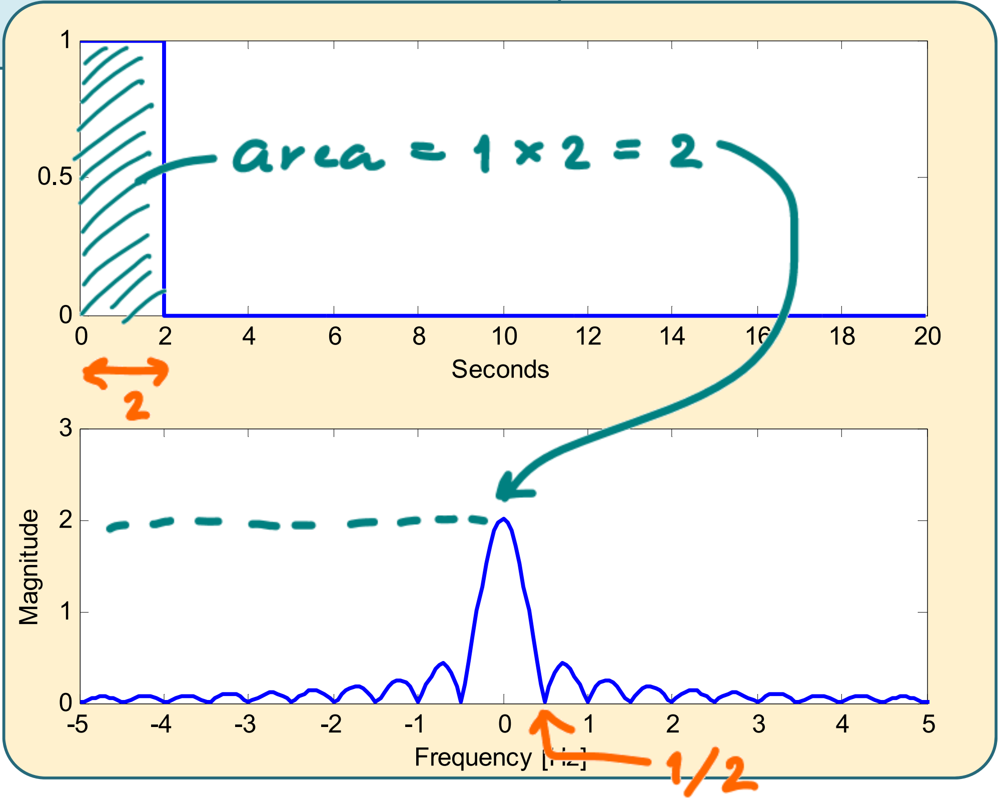
$$\int_{-\infty}^{\infty} 4\,\text{sinc}(4\pi f)\,df = \int_{-\infty}^{\infty} G(f)\,df = g(0) = 1$$

An Example for HW2

```
% specrect.m plot the spectrum of a square wave
close all
time=20;                % length of time
Ts=1/100;               % time interval between samples
t=0:Ts:(time-Ts);       % create a time vector
x=[t <= 2];             % rectangular pulse $1[0 \leq t \leq 2]$
plotspect(x,t)          % call plotspect to draw spectrum
xlim([-5,5])
```

$$x(t) = \begin{cases} 1, & 0 \leq t \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$

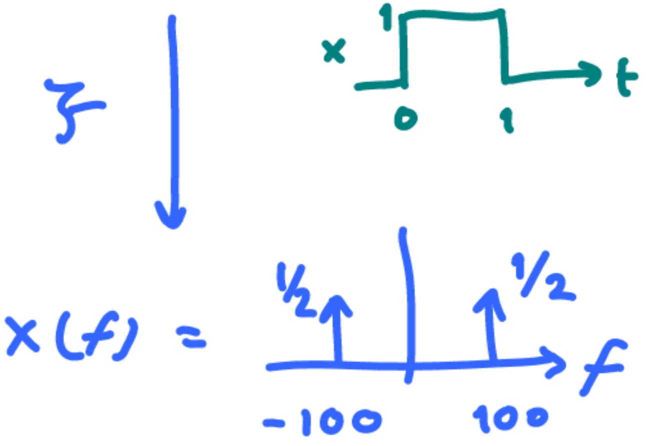
$|X(f)|$

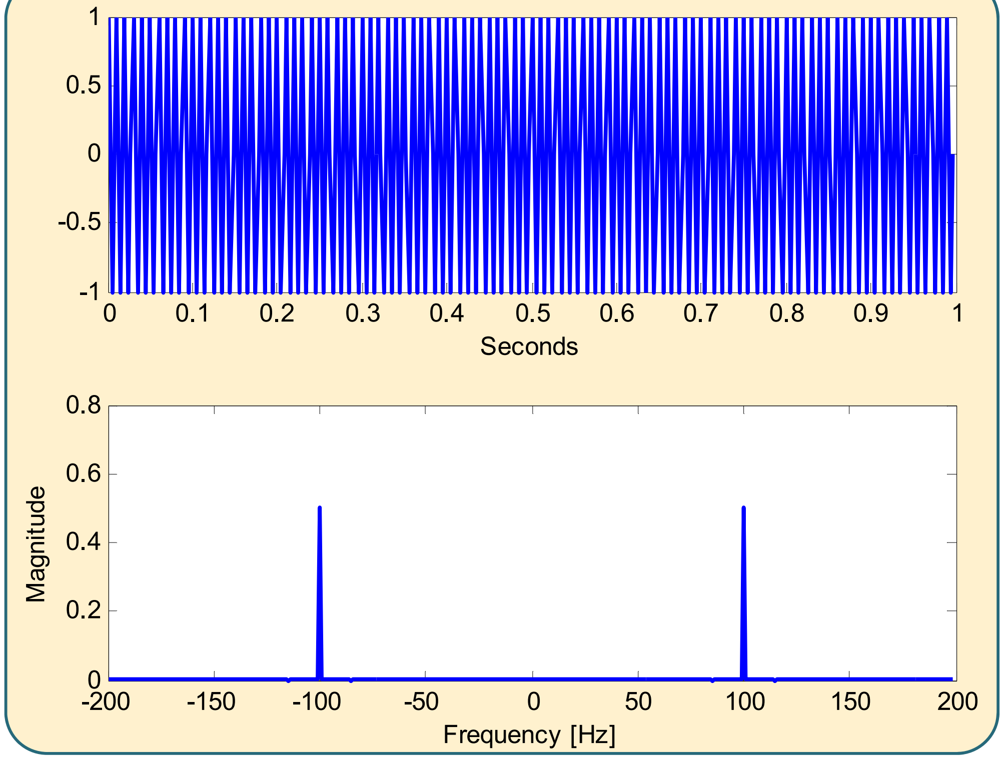


Another Example for HW3

```
close all
time=1;                 % length of time
Ts=1/400;               % time interval between samples
t=0:Ts:(time-Ts);       % create a time vector
x =cos(2*pi*100*t);% cosine signal
plotspect(x,t)          % call plotspect to draw spectrum
```

$$x(t) = \cos(2\pi(100)t)$$





Another Example for HW3

```
close all
time=1;                 % length of time
Ts=1/400;               % time interval between samples
t=0:Ts:(time-Ts);       % create a time vector
x =cos(2*pi*100*t).*(t <= 0.6).*(t >= 0.4);
                        % cosine pulse
plotspect(x,t)          % call plotspect to draw spectrum
```

$$x(t) = \begin{cases} \cos(2\pi(100)t), & 0.4 \le t \le 0.6, \\ 0, & \text{otherwise.} \end{cases}$$

$$= \cos(2\pi(100)t) \times$$

t

area = 0.2

$$\Big\downarrow \mathcal{F}$$

$$\left(\frac{1}{2}\delta(f-100) + \frac{1}{2}\delta(f-(-100))\right)$$

*